THE FOURTH CONDITION

A SPYGLASS DRAMA

STARRING

A.-C. CLAIRAUT AND J. LE ROND D'ALEMBERT

with special guest star Charles S. Hastings

PRODUCED AND DIRECTED BY

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Preface

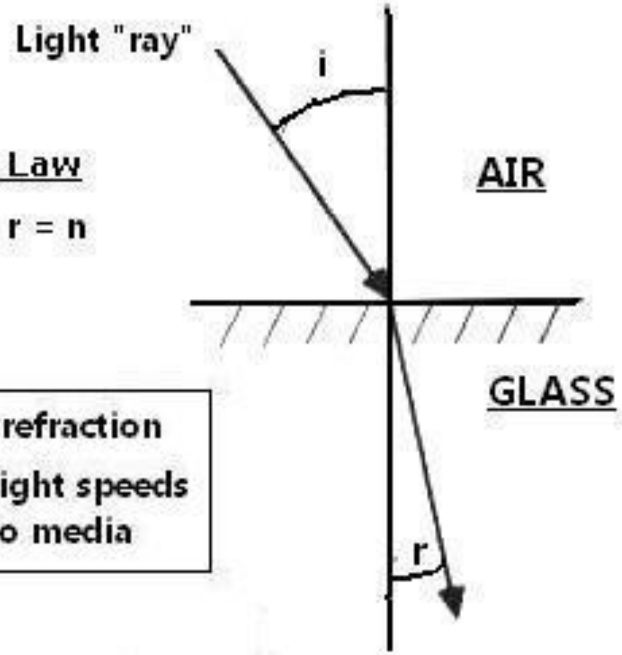
This PowerPoint is a revised version of a talk that was given at the January 12, 2021 meeting of the Amateur Astronomers Association of Princeton, NJ. It was given remotely due to the COVID-19 pandemic.

The author provided narration during the talk, which cannot be reproduced here. Hopefully the slides will be self-explanatory.

Most of the material is in the field of geometrical (i.e. "ray") optics, which is what optical designers use even though image quality is ultimately limited by physical or "wave" optics. Wave optics comes in when we talk about the size of the Airy disk of the Hastings objective.

The author hopes you will enjoy this material as much as he enjoyed putting it together from work done many years ago.

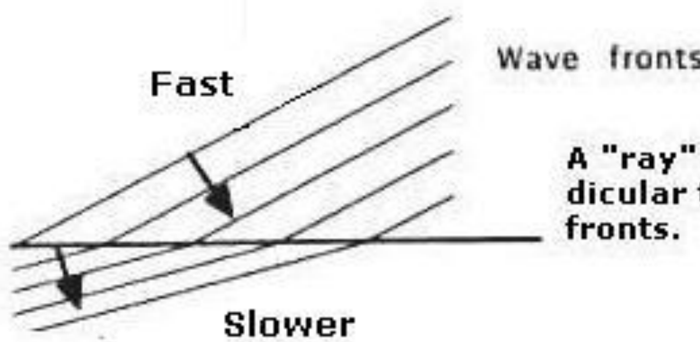
REFRACTION and GEOMETRICAL OPTICS



Snell's Law

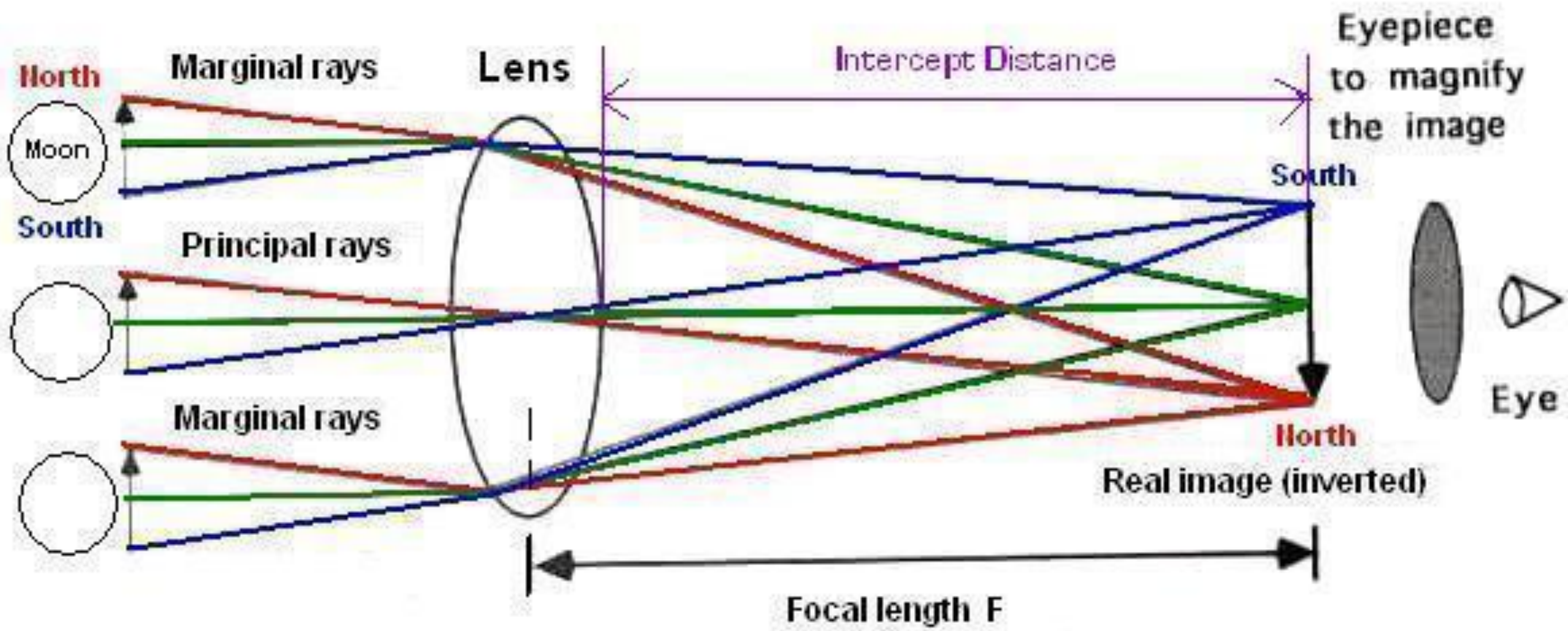
Sin i/Sin r = n

n = index of refraction
= ratio of light speeds in the two media



A "ray" is perpendicular to wave fronts.

REFRACTING TELESCOPE



OBJECT AT LARGE DISTANCE
Rays from all individual points come in parallel

[Adapted from H. Semat]

THE FIRST CONDITION

We want a certain focal length for our telescope. Long FL = big image (but a long tube!)

If we are using a simple lens, then we have the lens maker's formula:

$$1/F = (n - 1)\ (1/R_1 - 1/R_2)$$

R_1 and R_2 = radii of curvature of the first and second lens surfaces

(= positive radius;) = negative radius when light comes from the left

We have a piece of glass with a known index of refraction n. We specify the focal length we want, and one radius. The lens maker's formula then tells us the value of the other radius.

But it makes a very poor telescope.

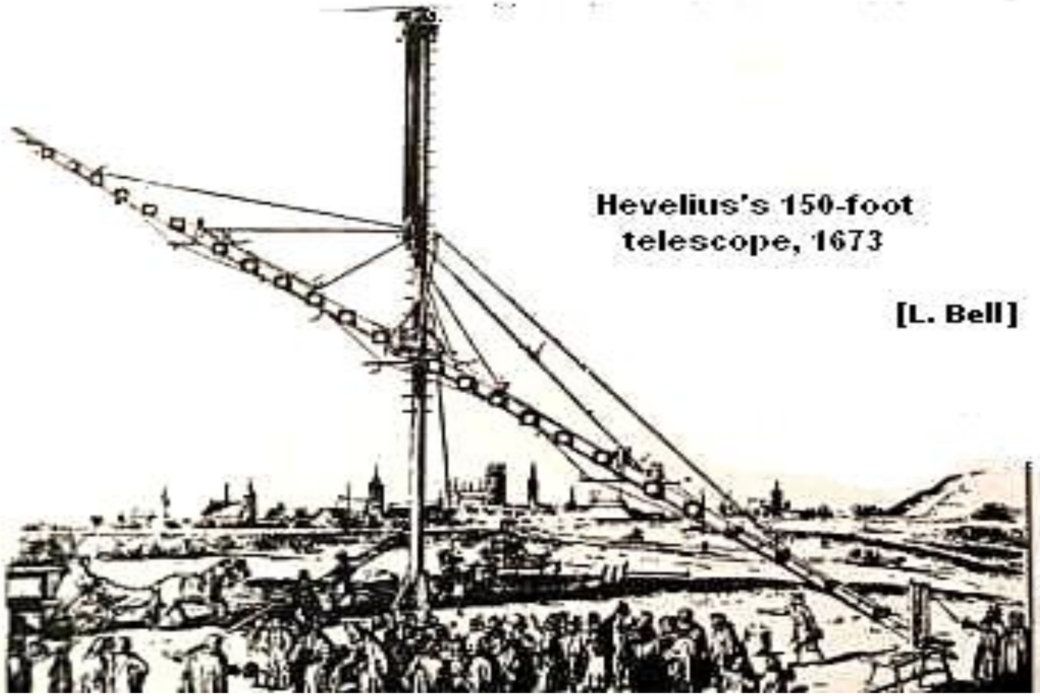
Because we have forgotten that n is not the same for all colors of light.

THE SOLUTION

... was to make the focal length so high that the color fringes got absorbed into the image.

For a given lens diameter (aperture), the color fringes remain the same physical size as the focal length increases, but the image gets bigger.

Extremely long and unwieldy telescopes were the result.



Hevelius's 150-foot telescope, 1673

[L. Bell]

This leads to ....

THE SECOND CONDITION

We want our telescope to be <u>achromatic</u>: as free as possible from chromatic aberration, when it is of a reasonable length.

Isaac Newton thought of a possible way.

NEWTON'S APPROACH

Newton knew in the late 1600's that different optical materials dispersed colors to different extents. That being the case, he thought it might be possible to combine two different lenses (one double convex, the other double concave) to cancel out the color and still have a net convergence of light to an image.

IT WAS THE RIGHT IDEA

.... but Newton did a hasty experiment which convinced him that all transparent materials had the same ratios of refraction to dispersion:

$$V = \frac{n(\text{yellow}) - 1}{n(\text{blue}) - n(\text{red})}$$

$$= \text{same for all materials?}$$

If this were the case ... which it isn't! ... you couldn't make a color-corrected lens that would have net converging power.

This led him to abandon the search for color correction (achromatism) in lenses, and to look for a better way.

So he invented the reflecting telescope.

? Suppose Newton had done his experiment right ... how long would it have taken to invent the "Newtonian" telescope ?

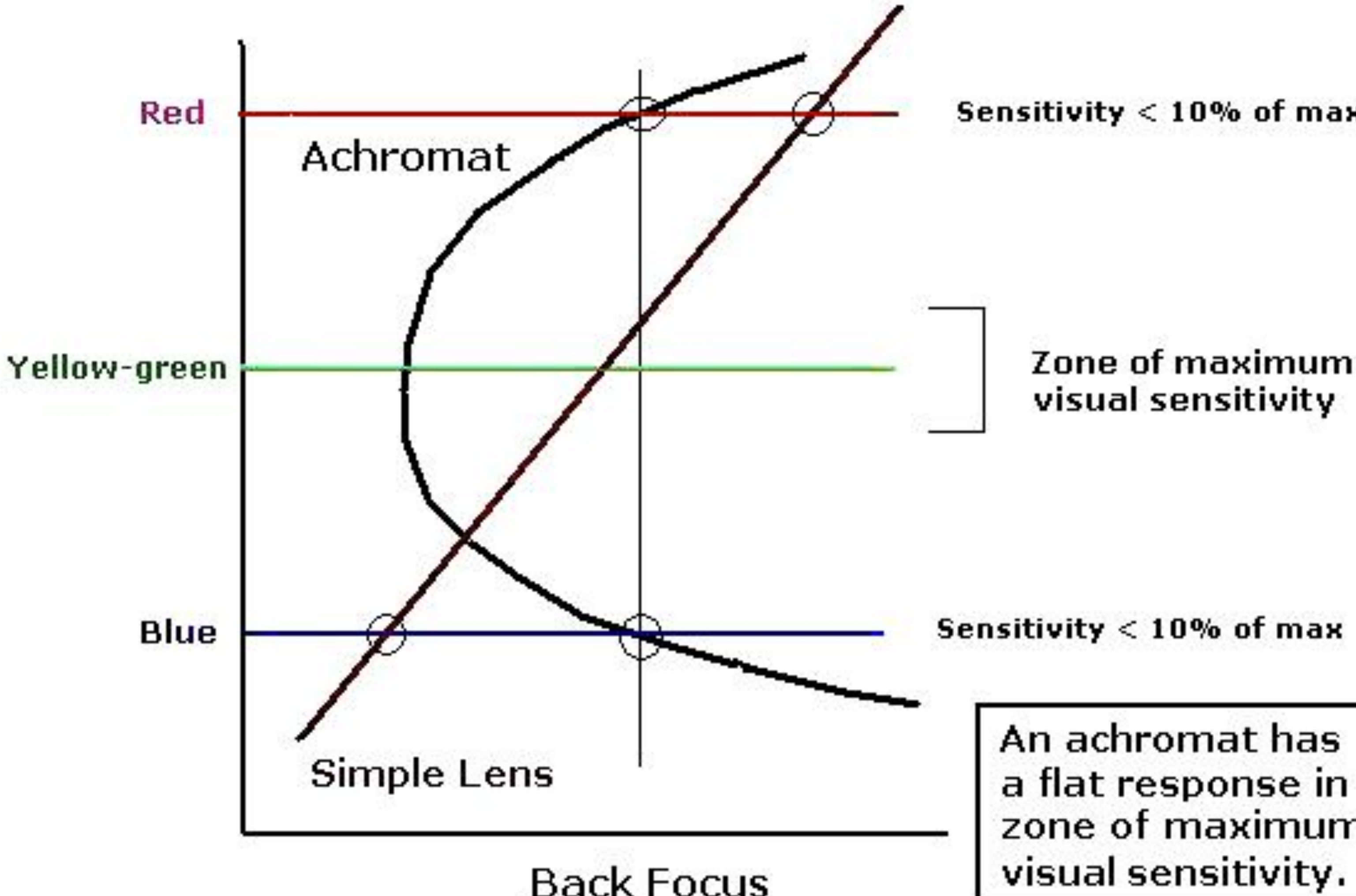
IF NEWTON SAYS IT CAN'T BE DONE.....

Do it anyway.

Chester Moor Hall designed and had made a working achromatic doublet lens in 1733.

Eventually the word got out that it could be done, and John Dollond began producing reasonably good achromats in 1758.

An achromat brings colors at the ends of the visual spectrum to the same focus.
A simple lens changes focus throughout the spectrum.



An achromat has a flat response in the zone of maximum visual sensitivity.

PERFECTION IS IMPOSSIBLE

The best that can be done with any two available glasses is to have two separate wavelengths focus to the same point on the axis.

Reason: irrationality of dispersion in all known pairs of glasses.

Change of refractive index with wavelength is not exactly proportional between the two glasses, over the visible spectrum.

Modern glasses can come closer than old ones to this ideal, but haven't reached it, also can be expensive and less durable (example: fluorite = calcium fluoride).

A 3-element lens can focus three wavelengths together ("apochromat").

MAKING AN ACHROMAT

Object: Make focal lengths the same for both red and blue light

* Choose 2 glasses with different V-nos. ("Crown" and "flint")

  Values for AAAP : V(crown) = 56.5
  V(flint) = 36.7

* Specify total focal length, F (first condition)

  For AAAP: F = 2313 mm

Eqn. 1: 1/F = 1/F(crown) + 1/F(flint)
Eqn. 2: F(flint)/F(crown) = -V(crown)/V(flint)

Solve 2 eqns. in 2 unknowns:

F(crown) = 811 mm

F(flint) = - 1249 mm

(Approximate values subject to later adjustments for other aberrations)

THE NEXT STEP

Now that the focal lengths are known for each element, we can apply the lens maker's formula to each one:

$$1/F = (n - 1)\,(1/R_1 - 1/R_2)$$

where n = index for yellow light

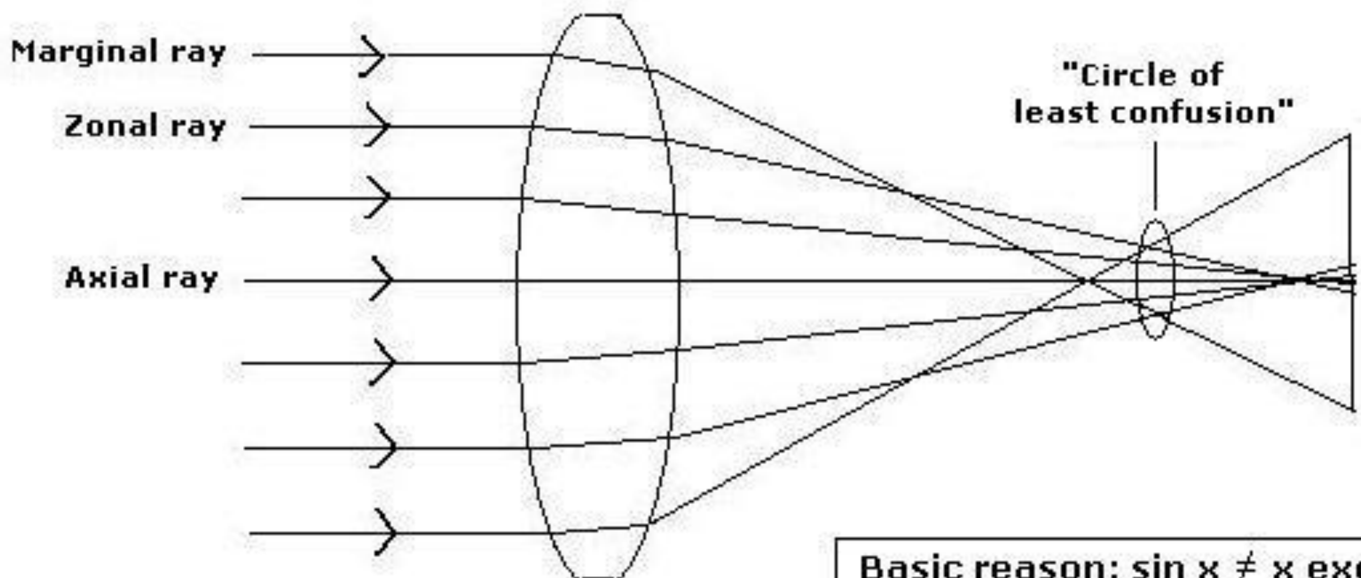
But: how are we to choose <u>which</u> values of R to use? The equation can be satisfied by an <u>infinite number</u> of radius pairs, all of which will give the correct focal length for that element, and the combination of the two elements will still be an achromat.

Answer: we can use these extra degrees of freedom to satisfy <u>two more conditions</u>.

THE THIRD CONDITION

Lenses with spherical surfaces
exhibit spherical aberration.

It can be minimized with very long focal lengths,
but we need to find another way.



Basic reason: sin x ≠ x except for very small x (x in radians)

CORRECTING SPHERICAL ABERRATION

Euler and others realized that with a two-element lens that corrected chromatic aberration as well as could be done, you could still adjust the curvatures of the lens surfaces in a way that would bring "paraxial" (i.e. nearly axial) and marginal rays to the same point in the image plane. You might still have a small amount of "zonal" aberration.

This approach "uses up" one of the two remaining degrees of freedom, because changing one radius fixed another one in order to retain achromatism.

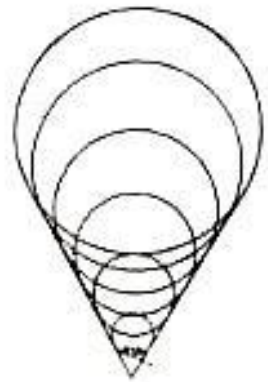
You had an infinite number of pairs of radii to choose from. After choosing one, you still had one degree of radius freedom left.

OPTIONS

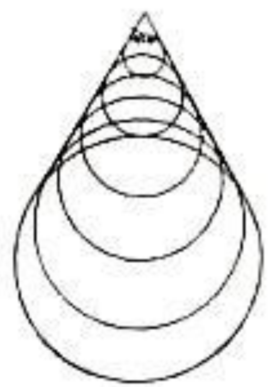
With focal length, chromatic aberration, and spherical aberration taken care of, we still have one degree of freedom left (since we have four radii to specify).

What shall we do?

- We could have equal internal radii - allows elements to be cemented **(small lenses only)**
- We could correct S.A. for two separate wavelengths (Gauss). Good for planets.
- We could have equal radii for the crown element (Clark). Only one tool needed.
- All of the above leave some degree of *coma*: comet-shaped images at edge.

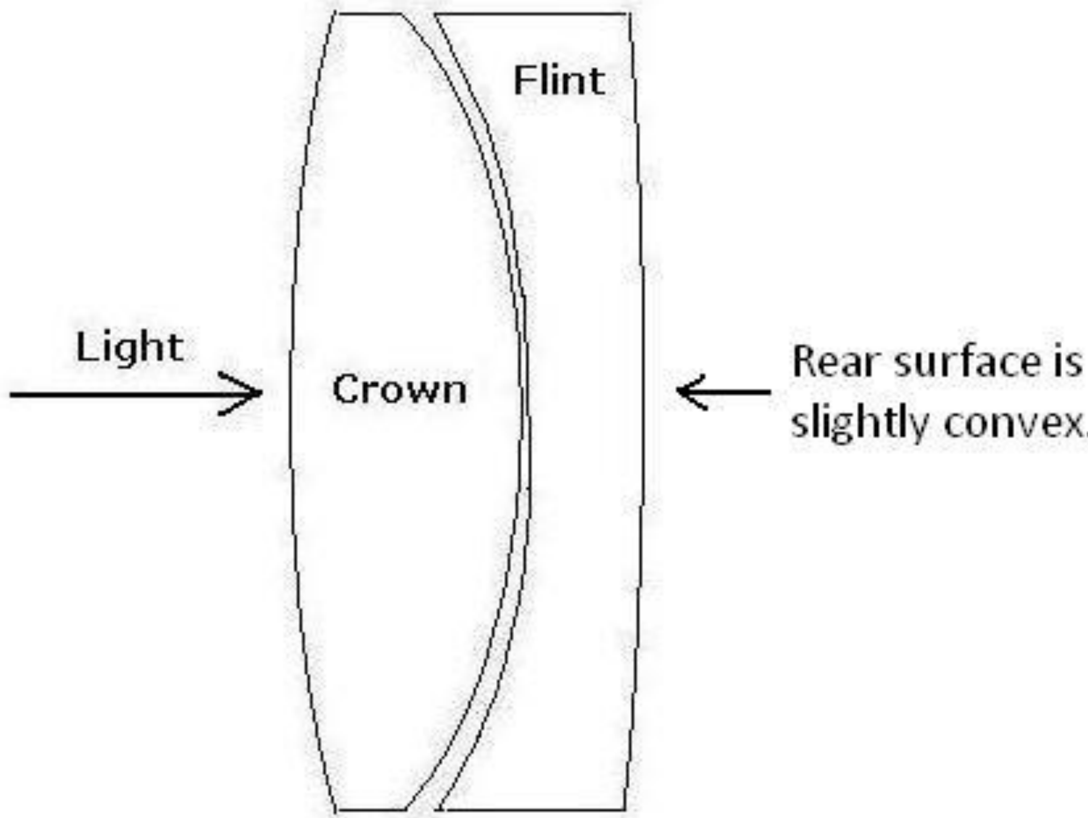


Outward Coma



Inward Coma

Coma increases from the center to the edge of the field.
Clairaut was the first to show how to remove it.



Although commonly called a "Fraunhofer" objective,
this form was first proposed by Clairaut.

[Mémoires de l'Académie Royale des Sciences, 1762, 615.]

THE FOURTH CONDITION - CORRECTING COMA

Intense competition between Clairaut and d'Alembert in 1763-1764; Clairaut published first.

Complicated formulas couldn't be easily used by most working opticians.

Fraunhofer (1820's) constructed low-coma lenses, probably without knowledge of the French papers. Trial and error ray tracing.

The Hastings 6-1/4 inch lens is a Fraunhofer (i.e. Clairaut) type with four different surface radii and a small central airspace.

Many others have published equations that give the same results as those of Clairaut and d'Alembert.

In 1887, Moser published a set of equations that are readily programmed.

Coma and the Sine Condition

To minimize coma, the objective must satisfy the "sine condition" (Abbe, 1873).

Sines of final slope angles proportional to sines of entering slope angles.

Before photography was extensive, designers were more concerned with field centers (planets, double stars).

Result: Some large refractors have significant coma (Sky & Telescope, March 1982, p. 302).

Estimate coma by "offense against the sine condition" (Conrady, p. 370). Should be less than +/- 0.0025 for visual use, much less for photography/astrometry.

Our Hastings objective has an OSC' of -0.00026 for the brightest part of the spectrum. It gives sharp star images out to the edges of low-power fields, for example with the Double Cluster in Perseus.

Moser's Equations in a Basic Program

```
10 REM  APLANATIC DOUBLET DESIGN
12 REM
14 INPUT "FRONT INDEXES";A,B,C
16 INPUT "REAR INDEXES ";D,E,F
18 INPUT "FOCAL LENGTH ";FL
20 PRINT
22 G=B/(B-1): H=E/(E-1)
24 I=(B-1)/(C-A): J=(E-1)/(F-D)
26 K=I^2: L=I^3: M=J^2: N=J^3
28 O=(3-2/G)*I: P=(3-2/H)*J
30 Q=(3*G-1)*K
32 R=(8-4/H)*I*J-(3*H-1)*M
34 S=(G^2)*L-(5-2/H)*K*J
36 T=(4*H-1)*I*M-(H^2)*N
38 U=(2-1/G)*I: V=(2-1/H)*J
40 W=G*K-(3-1/H)*I*J+H*M
42 X=(U^2)*P-(V^2)*O
44 Y=2*U*W*P-(V^2)*Q+U*V*R
46 Z=(W^2)*P-(V^2)*(S+T)+V*W*R
48 A1=(Y-SQR(Y^2-4*X*Z))/(2*X)
50 A2=(U*A1)/V-W/V
52 R1=((I-J)/A1)*FL
54 R2=((I-J)/(A1-(I*(G-1))))*FL
56 R3=((I-J)/A2)*FL
58 R4=((I-J)/(A2+(J*(H-1))))*FL
60 PRINT "R1 = ";R1
62 PRINT "R2 = ";R2
64 PRINT "R3 = ";R3
66 PRINT "R4 = ";R4
68 END
```

Provisos

Best results at above f/10. Always ray-trace with realistic thicknesses.

Excel spreadsheet gives same results.

J. Church, Sky & Telescope 68:450 (1984)

For a flint-in-front design, in Line 48 change the sign before "SQR" to + and enter the flint indexes first.

Alexis Claude Clairaut

1713-1765
[Wikipedia]

J. le Rond d'Alembert
1717-1783
[Wikipedia]

THE CLAIRAUT - d'ALEMBERT RIVALRY

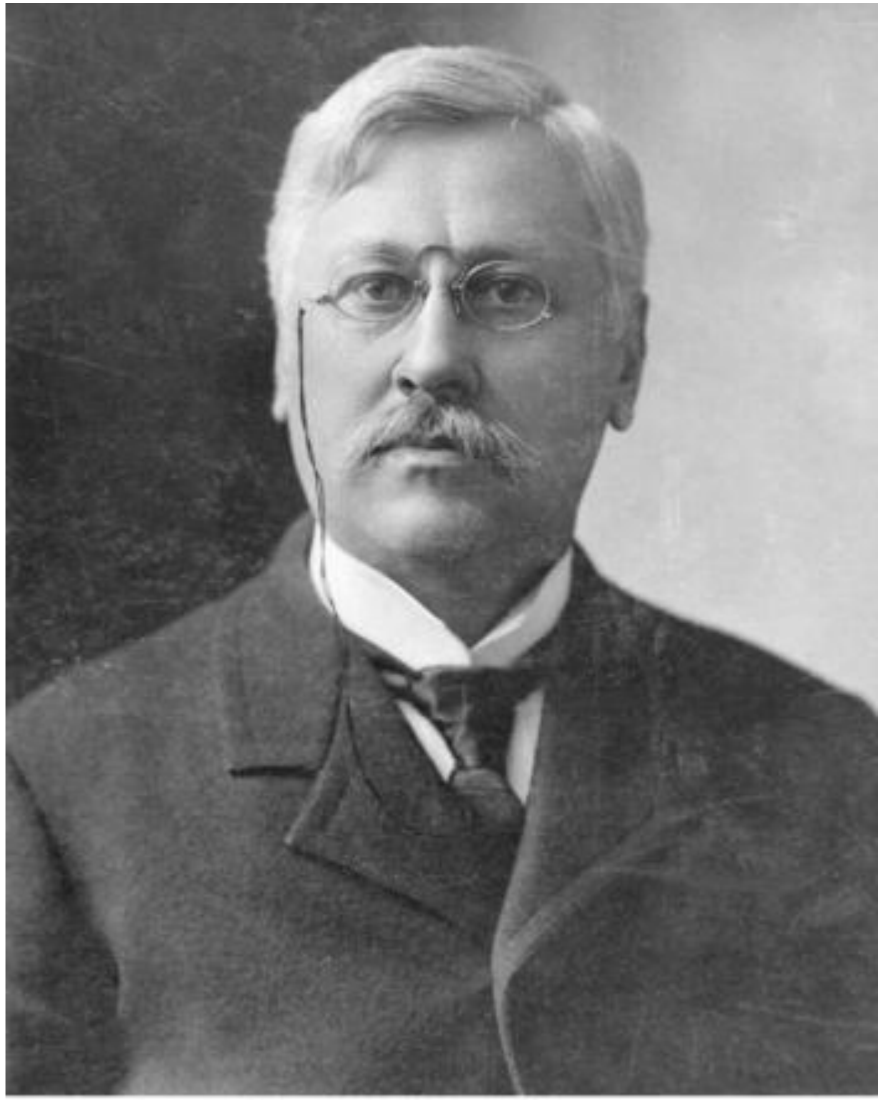
Clairaut and d'Alembert were intense rivals in the mid-1700's.

Both were highly talented mathematicians and rarely attended each other's public lectures.

Clairaut was the first to accurately predict the return of Halley's comet in 1759, followed by d'Alembert's detailed analysis after the fact.

Clairaut published his design methods first. d'Alembert was slightly later due to delays caused by the Seven Years' War in Europe.

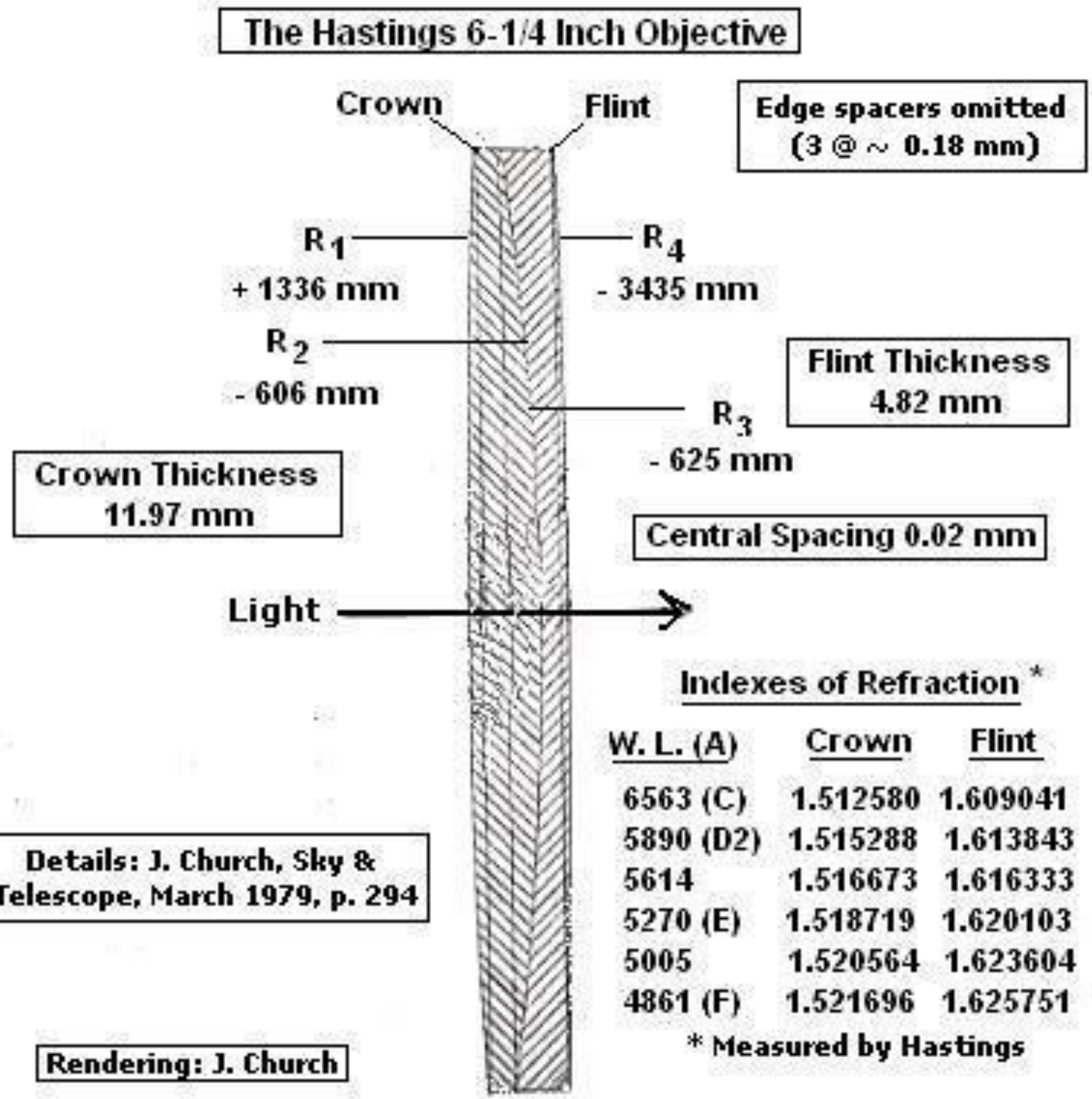
Clairaut died soon afterwards and d'Alembert went on to other areas.



Charles S. Hastings (1848-1932)

Courtesy of his granddaughter, Mrs. Thomas Lowry.

Mrs. Lowry and her son and grandchildren came to our May 1979 meeting in Peyton Hall.



Indexes of Refraction *

W. L. (A)	Crown	Flint
6563 (C)	1.512580	1.609041
5890 (D2)	1.515288	1.613843
5614	1.516673	1.616333
5270 (E)	1.518719	1.620103
5005	1.520564	1.623604
4861 (F)	1.521696	1.625751

* Measured by Hastings

The Hastings 6-1/4 inch Objective in its cell by John Byrne

Spot Diagrams for the 6-1/4 inch Hastings Objective

Spot diagrams produced by advanced optics software show what an observer may actually see in an eyepiece.

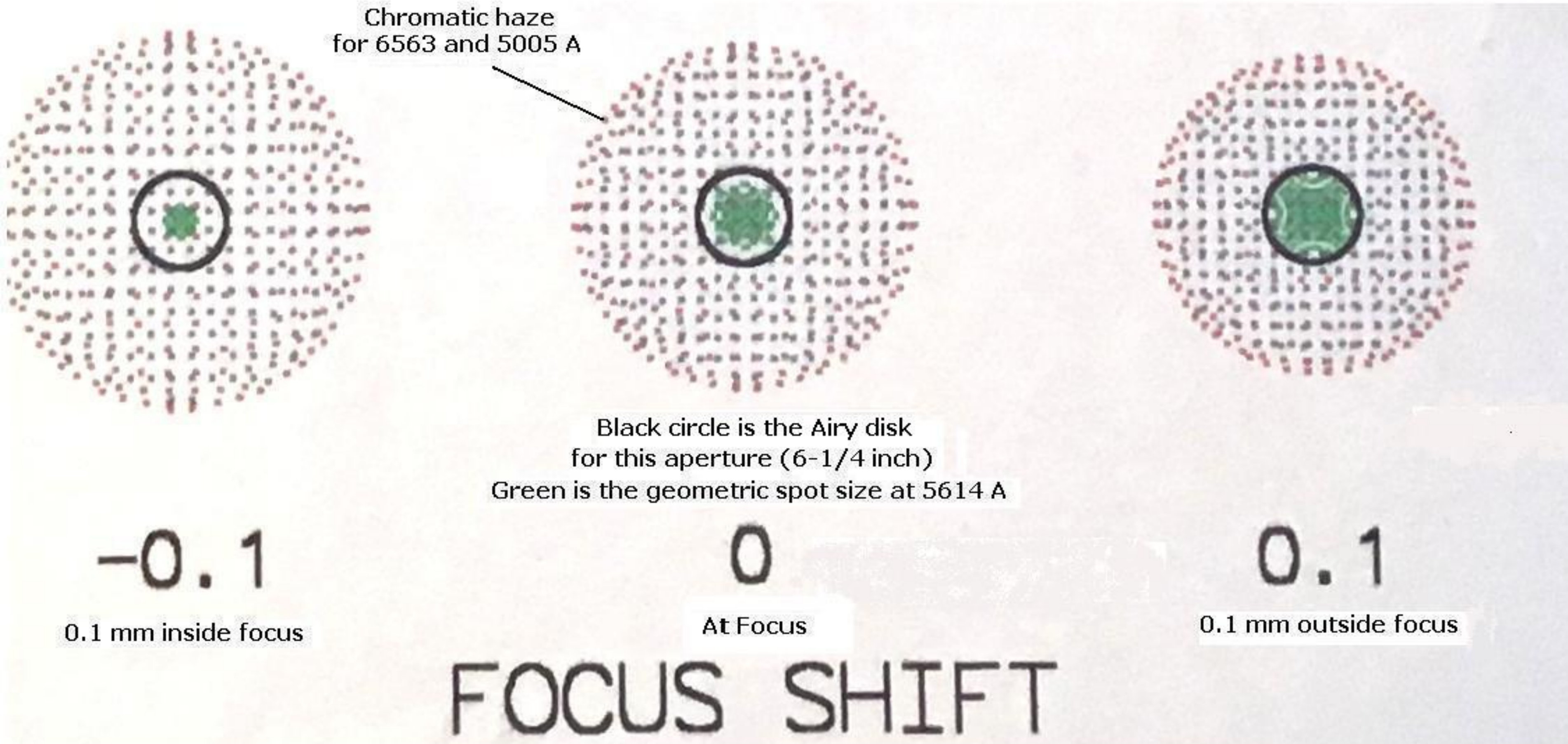
When a star is observed with an achromat, a faint color haze may be seen around the "Airy disk," which is produced by diffraction effects at the rim of the objective.

The larger the lens, the smaller the Airy disk will be.

When an objective is highly corrected, the geometric spot will be smaller than the Airy disk. But the star will look no smaller than the disk. In other words, the system is "diffraction limited," i.e., as good as possible.

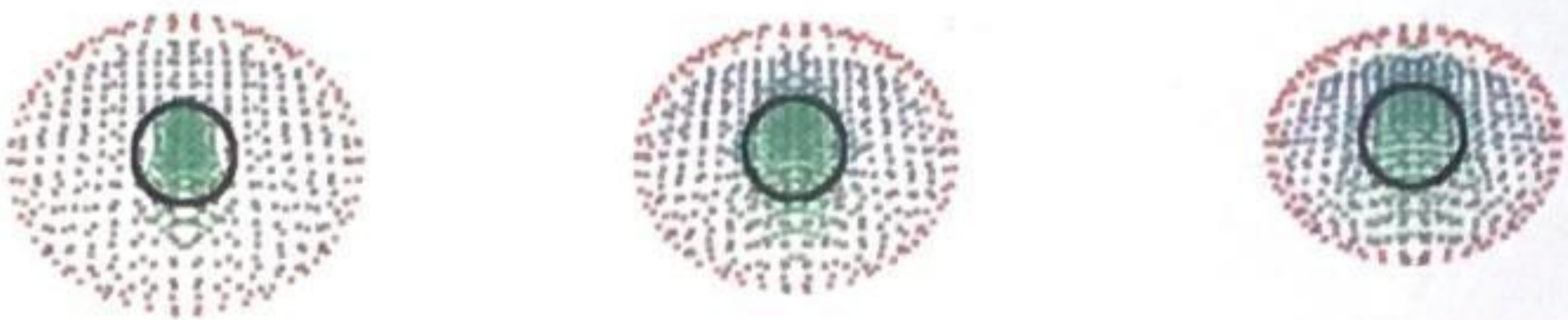
The Hastings objective is in this category for the part of the visual spectrum to which the eye is most sensitive.

The following slides show spot diagrams for both on-axis and off-axis performance. The off-axis spots are nearly as good as the on-axis spots. This lens is nearly free of coma.



AAAP Hastings Objective Spot Diagram On Axis (OSLO, jac data). Diagrams courtesy of Matt Considine.
(OSLO is short for Optics Software for Layout and Optimization, Lambda Research Corporation, Littleton, MA)

Slight astigmatism
Very slight coma



0.1 mm inside focus | At Focus | 0.1 mm outside focus

AAAP Hastings Objective 0.5 degrees off-axis
(Full Field 1.0 degrees)

OSLO w/jac data

Diagrams courtesy of Matt Considine.

What if Hastings Had Designed the AAAP Objective Using Moser's Formulas ?

	Hastings Radii	Moser Radii *
R1	1336 mm	1417 mm
R2	- 606	- 592
R3	- 625	- 610
R4	- 3435	- 3002

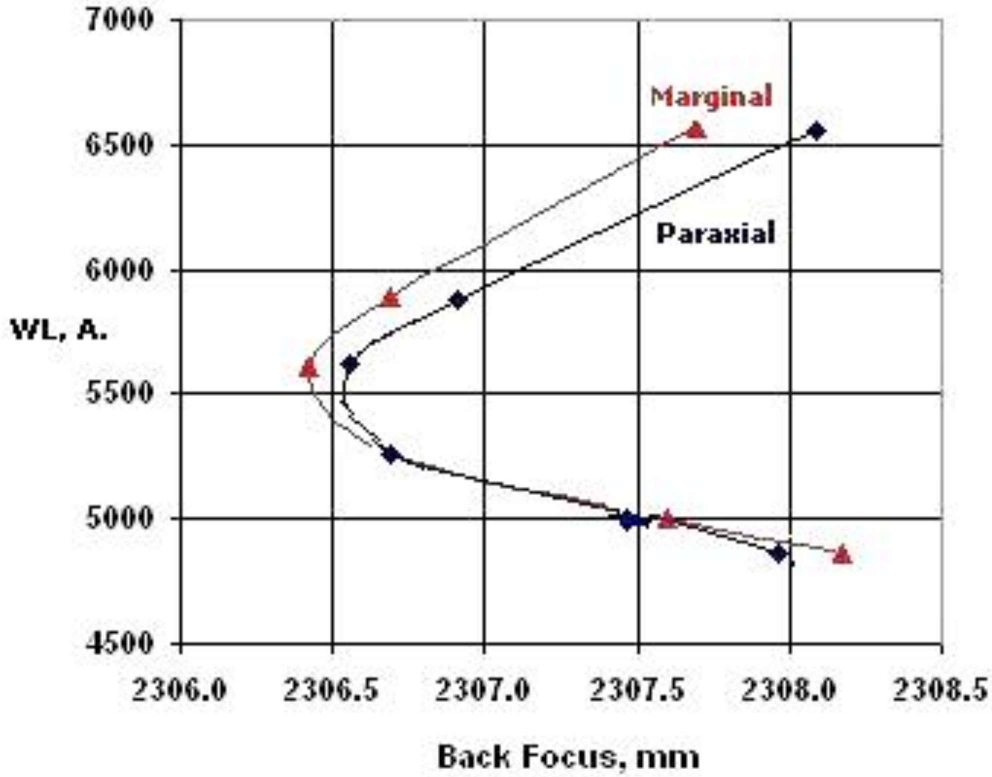
Both objectives are well-corrected for spherical aberration and coma. The Moser radii give slightly better corrections, but the difference would not be noticed.

* **Computed for the Hastings glasses with focal length 2313 mm and achromatized for 6563, 5614, and 4861 A.**
Moser's formulas give the same results as those of Clairaut and d'Alembert.

Ray-traced with the Hastings thicknesses and space.

The color curves for the "redesigned" Hastings 6-1/4" objective are <u>nearly an exact match to the original lens</u> except for a slightly longer back focus. We do not know the details of how Hastings actually designed this lens.

Color Curves for Hastings 6-1/4" Objective
As "Redesigned" by Clairaut/d'Alembert/Moser



For the curves for the actual Hastings lens, see <u>Sky & Telescope</u> for March 1979, p. 299.

Other Objectives Designed by Hastings

[J. Church, Sky & Telescope March 1982, p. 302]

1. The Johns Hopkins 9.4-Inch (made by Hastings ca. 1880) (Sidereal Messenger Vol. 2 p. 41, 1883)

   Flint in front, S.A. zero at 5550 A, achromatized for marginal rays at C and 5005 A, but has OSC +0.0013 and therefore some outward coma. This lens seems to have disappeared.

2. The Sproul Observatory 24-inch (made by Brashear, 1911)

   Flint in front. Radii supplied by Hastings in 1924 are probably in error as the lens is much better than these would indicate. It has residual coma (OSC ~ +0.0008).

   Swarthmore closed the observatory in 2017 and the telescope has been moved to Bentonville, Arkansas to be restored and used for public outreach.

Hastings was a long-time consultant to Brashear and designed other large refractor objectives.

Hastings Triplet Hand Lens

Other Possible Aberrations

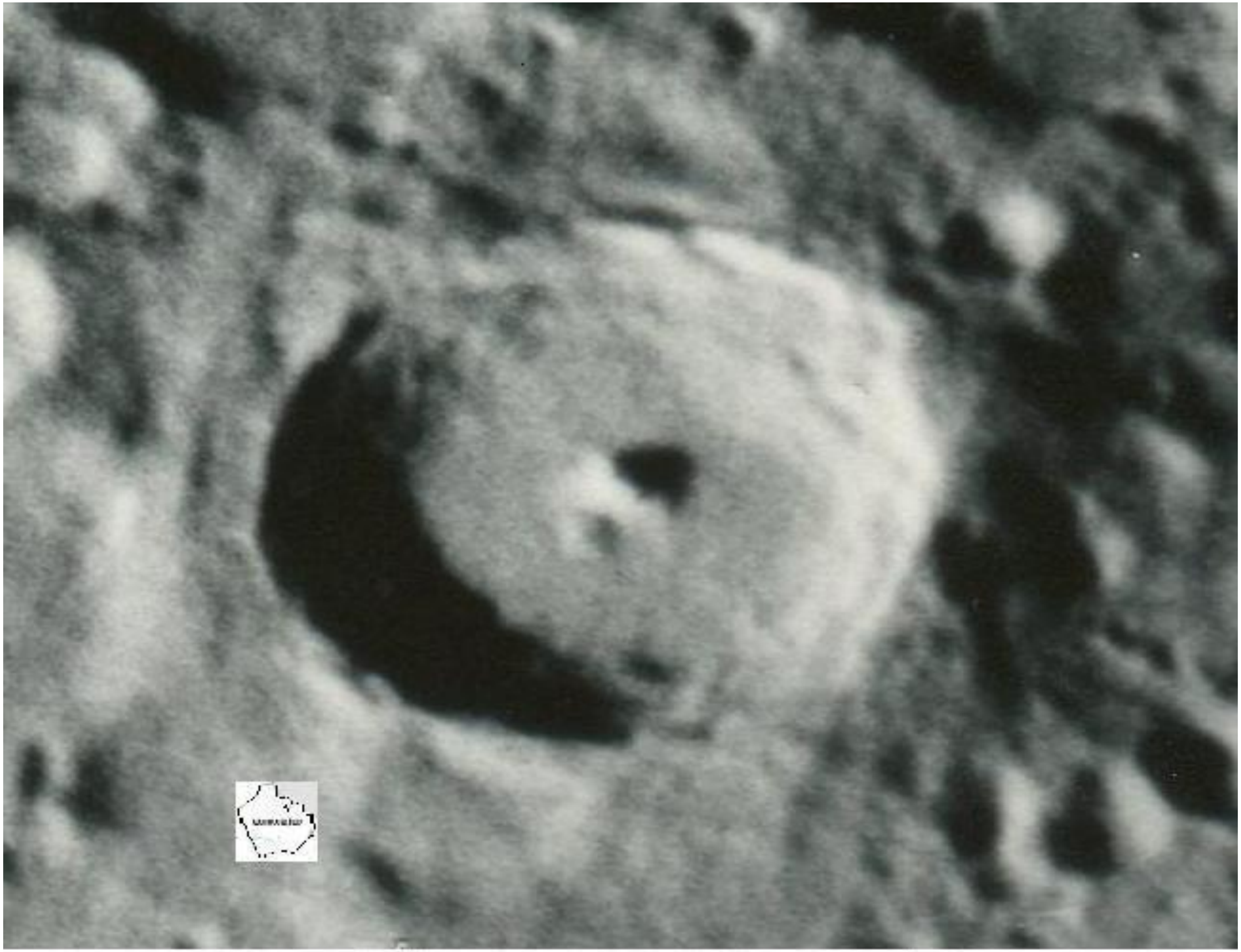
1. Lateral color (chromatic difference of magnification)
   - different image sizes for different colors; more likely when elements are spaced apart (large Clark lenses)

2. Astigmatism ("not point")
   - first studied by Clairaut, who drew detailed diagrams
   - ellipses and lines off-axis 
   - present but not a major issue in long-focus refractors
   - significant concern at small f/ratios with large fields

3. Distortion
   - Pincushion  Barrel 
   - not usually an issue in long-focus refractors

4. Field Curvature
   - center in focus, edge out of focus
   - not significant in long-focus refractors
   - calculate via Petzval sum; needs all radii and indices but makers generally don't divulge these trade secrets

Lunar Crater Clavius, taken with the Hastings-Byrne 6-1/4 inch.



Lunar Crater Tycho, taken with the Hastings-Byrne refractor.
Inset: West Windsor Township to approximate scale.

[Photo: J. Church]

Science vs. Trade Secrets

An Old Dilemma

Scientists such as Clairaut and d'Alembert worked for fame and the thrill of advancing knowledge of how optics really works

Their work was openly published and could have had many applications

Aside from patents, opticians have generally kept their methods secret

Alvan Clark & Sons made many good telescopes, but left very few records

Today's makers provide color curves, glass types, and spot diagrams but rarely enough data to allow analysis of all the aberrations

References and Credits

D. Jacobs, Fundamentals of Optical Engineering

G. Boutry, Instrumental Optics

L. Bell, The Telescope

H. Semat, Fundamentals of Physics

Sky & Telescope magazine

A. Conrady, Applied Optics and Optical Design, Part One

Special thanks to Ira Polans for assistance with Zoom presentations.